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About A LEVEL

MATHEMATICS
Paper 4
(TOPICAL)

period	2010 to 2025
contents	June & November, P4, Worked Solutions
form	Topic By Topic
compiled for	AS & A Levels
special features	Learning Corner

About Learning Corner

This column will further enrich students with the extra information and knowledge provided. It improves the students' ability to solve problems of similar style. Other important guidances are also stated.

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- NEW SYLLABUS

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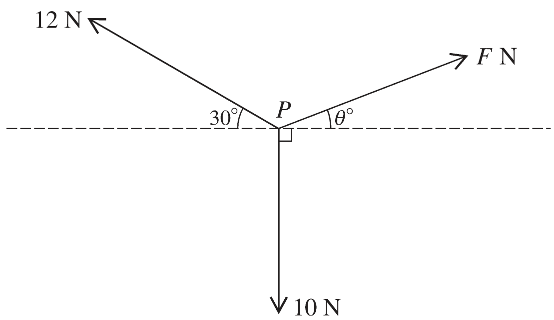
REVISION

- June / November 2025 Paper 4

Topic 1

Forces and Equilibrium

Question 1



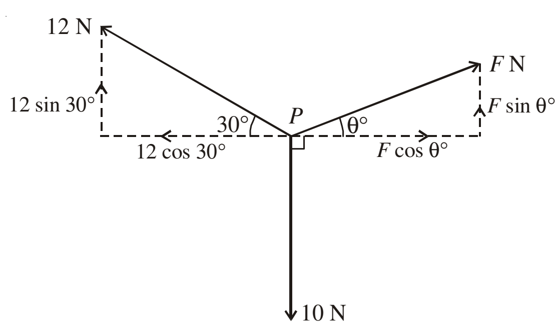
The three coplanar forces shown in the diagram act at a point P and are in equilibrium.

(i) Find the values of F and θ. [6]

(ii) State the magnitude and direction of the resultant force at P when the force of magnitude 12 N is removed. [2]

[J11/P4/Q4]

Suggested Solution:



(i) Resolving the forces horizontally,

$\rightarrow$: $F\cos\theta = 12\cos 30°$

$F\cos\theta = 10.392$ ············(i)

resolving the forces vertically,

$\uparrow$: $F\sin\theta + 12\sin 30° = 10$

$F\sin\theta = 10 - 12\sin 30°$

$F\sin\theta = 4$ ············(ii)

divide eq. (ii) by eq. (i),

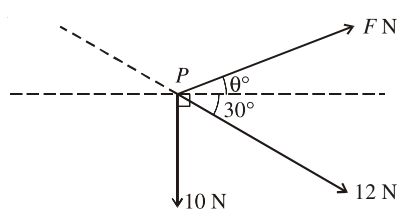
$$\frac{F\sin\theta}{F\cos\theta} = \frac{4}{10.392}$$

$\Rightarrow$ $\tan\theta = 0.3849$ $\Rightarrow$ $\theta = 21.1°$ **(Ans).**

substitute $\theta = 21.1°$ into equation (ii),

$F\sin 21.1° = 4$

$$\Rightarrow \quad F = \frac{4}{\sin 21.1°} = 11.1 \text{ N} \quad \textbf{(Ans).}$$



(ii) When the force of 12 N is removed, the resultant force at P is of magnitude 12 N with direction opposite to the previously applied 12 N force, i.e. 30^{o} clockwise from positive x-axis.

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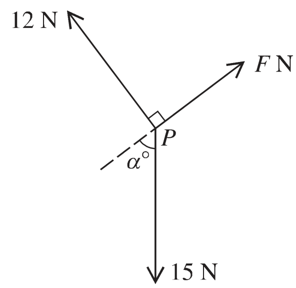
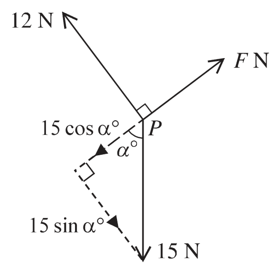
Question 2



Three coplanar forces of magnitudes F N, 12 N and 15 N are in equilibrium acting at a point P in the directions shown in the diagram. Find α and F. [4]

[J12/P4/Q2]

Suggested Solution:



Resolving the forces along and perpendicular to F N force,

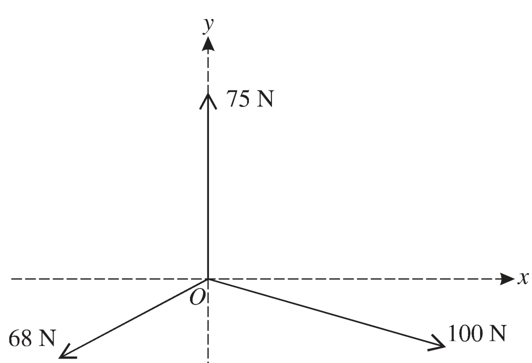
↙ : $F = 15\cos\alpha$ ·········(1)

↘ : $12 = 15\sin\alpha \;\Rightarrow\; \sin\alpha = \dfrac{12}{15} \;\Rightarrow\; \alpha = 53.1^\circ$ **(Ans).**

substitute $\alpha = 53.1^\circ$ into equation (1),

$F = 15\cos 53.1^\circ = 9$ N **(Ans).**

Question 3



Three coplanar forces of magnitudes 68 N, 75 N and 100 N act at an origin O, as shown in the diagram. The components of the three forces in the positive x-direction are – 60 N, 0 N and 96 N, respectively.

Find

(i) the components of the three forces in the positive y-direction, [3]

(ii) the magnitude and direction of the resultant of the three forces. [4]

[N12/P4/Q4]

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Suggested Solution:

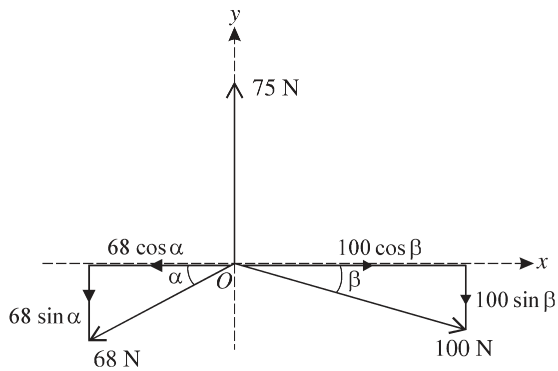
(i) Resolving the forces horizontally and vertically, we have,

$\rightarrow$: $-68\cos\alpha = -60$

$$\Rightarrow \quad \cos\alpha = \frac{60}{68} \quad \Rightarrow \quad \alpha = 28.07° \approx 28.1°$$

$\uparrow$: $100\cos\beta = 96$

$$\Rightarrow \quad \cos\beta = \frac{96}{100} \quad \Rightarrow \quad \beta = 16.26° \approx 16.3°$$



components of the three forces in positive y-direction are,

Y_1: $-68\ \sin\alpha = -68\sin(28.07°) = -32.0$ N **(Ans).**

Y_2: 75 N **(Ans).**

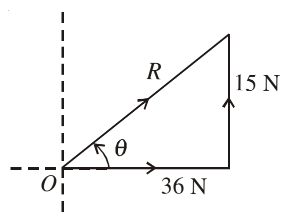
Y_3: $-100\ \sin\beta = -100\ \sin(16.26°) = -28.0$ N **(Ans).**

(ii) $\sum F_x = -60 + 96 = 36$ N

$\sum F_y = -32 + 75 - 28 = 15$ N

$\therefore$ Resultant force, $R = \sqrt{F_x^{\ 2} + F_y^{\ 2}}$

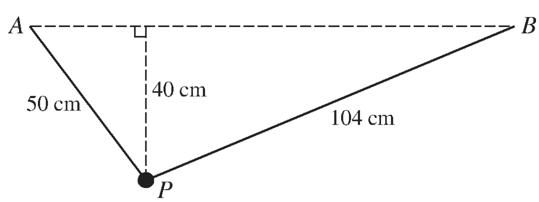
$$= \sqrt{(36)^2 + (15)^2} = 39 \text{ N} \quad \textbf{(Ans).}$$



for direction,

$$\theta = \tan^{-1}\left(\frac{15}{36}\right) = 22.6° \text{ with positive direction of } x\text{-axis} \quad \textbf{(Ans).}$$

Question 4



A particle P of mass 2.1 kg is attached to one end of each of two light inextensible strings. The other ends of the strings are attached to points A and B which are at the same horizontal level. P hangs in equilibrium at a point 40 cm below the level of A and B, and the strings PA and PB have lengths 50 cm and 104 cm respectively (see diagram). Show that the tension in the string PA is 20 N, and find the tension in the string PB. [5]

[J13/P4/Q3]

Suggested Solution:

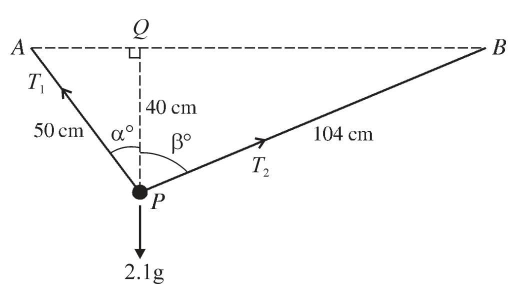
Let T_1 and T_2 be the tensions in strings PA and PB respectively,

In ΔAPQ, $\cos\alpha = \dfrac{40}{50} \quad \Rightarrow \quad \alpha = 36.87°$

In ΔBPQ, $\cos\beta = \dfrac{40}{104} \quad \Rightarrow \quad \beta = 67.38°$

$\therefore$ $A\hat{P}B = \alpha + \beta$

$$= 36.87° + 67.38° = 104.25°$$



applying Lami's theorem,

$$\frac{T_1}{\sin(180° - \beta°)} = \frac{21}{\sin 104.25°} \quad \Rightarrow \quad T_1 = \frac{21}{\sin 104.25°} \times \sin 112.62° = 20$$

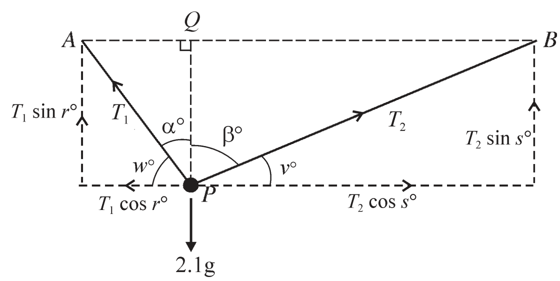
$\therefore$ tension in $PA = 20$ N **(Shown).**

applying Lami's theorem again for T_2,

$$\frac{T_2}{\sin(180° - \alpha°)} = \frac{21}{\sin 104.25°} \quad \Rightarrow \quad T_2 = \frac{21}{\sin 104.25°} \times \sin 143.13° = 13$$

$\therefore$ tension in $PB = 13$ N **(Ans).**

Alternative Method:



From figure, $\hat{w} = 90° - \alpha° = 53.13°$

$\hat{v} = 90° - \beta° = 22.62°$

Resolving the forces horizontally at P,

$\rightarrow$: $T_2 \cos v° = T_1 \cos w°$

$$T_2 = \frac{T_1 \cos w°}{\cos v°} \quad \Rightarrow \quad T_2 = \frac{T_1 \cos 53.13°}{\cos 22.62°} \quad \Rightarrow \quad T_2 = 0.65\ T_1$$

resolving vertically,

$\uparrow$: $T_1 \sin w° + T_2 \sin v° = 21$

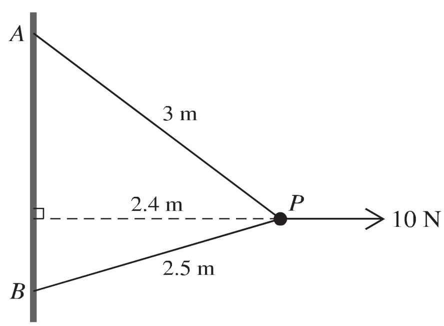
$T_1 \sin 53.13° + T_2 \sin 22.62° = 21$

$0.8\ T_1 + 0.385(0.65\ T_1) = 21$

$1.05 T_1 = 21 \quad \Rightarrow \quad T_1 = 20$ **(Shown).**

$\therefore$ $T_2 = 0.65(20) = 13$ N **(Ans).**

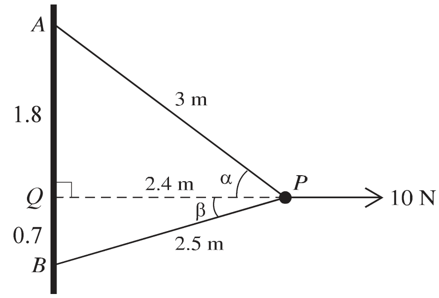
Question 5



A and *B* are fixed points of a vertical wall with *A* vertically above *B*. A particle *P* of mass 0.7 kg is attached to *A* by a light inextensible string of length 3 m. *P* is also attached to *B* by a light inextensible string of length 2.5 m. *P* is maintained in equilibrium at a distance of 2.4 m from the wall by a horizontal force of magnitude 10 N acting on *P* (see diagram). Both strings are taut, and the 10 N force acts in the plane *APB* which is perpendicular to the wall. Find the tensions in the strings. [6]

[J14/P4/Q3]

Suggested Solution:



In ΔAPQ, $\cos\alpha = \dfrac{2.4}{3}$, $\sin\alpha = \dfrac{\sqrt{3^2 - 2.4^2}}{3} = \dfrac{1.8}{3}$

In ΔBPQ, $\cos\beta = \dfrac{2.4}{2.5}$, $\sin\beta = \dfrac{\sqrt{2.5^2 - 2.4^2}}{2.5} = \dfrac{0.7}{2.5}$

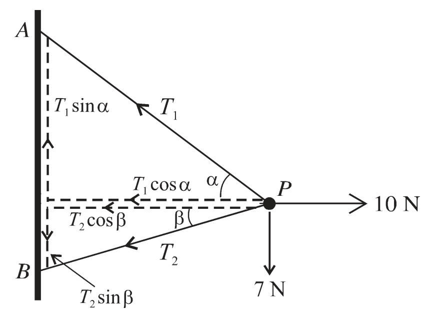
Resolving the forces horizontally at P,

$\rightarrow$: $T_1\cos\alpha + T_2\cos\beta = 10$

$$T_1\left(\frac{2.4}{3}\right) + T_2\left(\frac{2.4}{2.5}\right) = 10$$

$$0.8T_1 + 0.96T_2 = 10 \quad\Rightarrow\quad T_1 = \frac{1}{0.8}(10 - 0.96T_2) \cdots\cdots\cdots\cdots(1)$$

resolving the forces vertically at P,



$\uparrow$: $T_1\sin\alpha = T_2\sin\beta + 7$

$$T_1\left(\frac{1.8}{3}\right) = T_2\left(\frac{0.7}{2.5}\right) + 7$$

$$0.6T_1 = 0.28T_2 + 7 \quad\Rightarrow\quad T_1 = \frac{1}{0.6}(0.28T_2 + 7) \cdots\cdots\cdots\cdots(2)$$

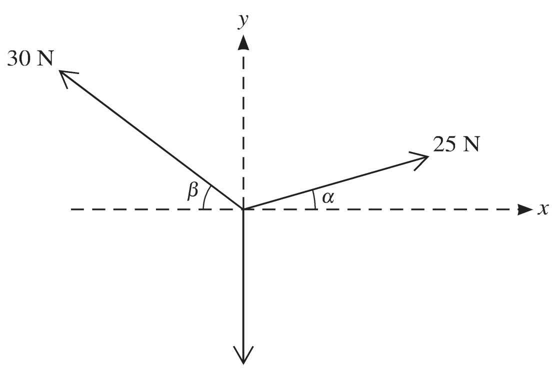
solving equations (1) & (2) simultaneously,

$$\frac{1}{0.8}(10 - 0.96T_2) = \frac{1}{0.6}(0.28T_2 + 7)$$
$$0.6(10 - 0.96T_2) = 0.8(0.28T_2 + 7)$$
$$6 - 0.576T_2 = 0.224T_2 + 5.6$$
$$0.8T_2 = 0.4$$
$$T_2 = 0.5$$

substitute $T_2 = 0.5$ in equation (1), $T_1 = \dfrac{1}{0.8}\big(10 - 0.96(0.5)\big) = \dfrac{1}{0.8}(9.52) = 11.9$

$\therefore$ $T_1 = 11.9$ N, $T_2 = 0.5$ N **(Ans).**

Question 6



Three coplanar forces act at a point. The magnitudes of the forces are 20N, 25N and 30N, and the directions in which the forces act are as shown in the diagram, where $\sin\alpha = 0.28$ and $\cos\alpha = 0.96$, and $\sin\beta = 0.6$ and $\cos\beta = 0.8$.

(i) Show that the resultant of the three forces has a zero component in the x-direction. [2]

(ii) Find the magnitude and direction of the resultant of the three forces. [2]

(iii) The force of magnitude 20N is replaced by another force. The effect is that the resultant force is unchanged in magnitude but reversed in direction. State the magnitude and direction of the replacement force. [1]

[N14/P4/Q2]

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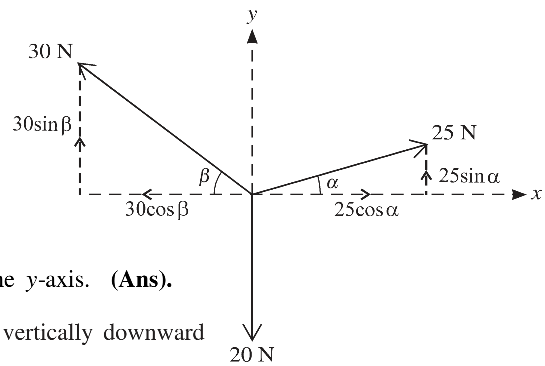
Suggested Solution:



(i) Resolving the forces horizontally,

$$\sum F_x = 25\cos\alpha - 30\cos\beta$$
$$= (25)(0.96) - (30)(0.8) = 24 - 24 = 0 \quad \textbf{(Shown).}$$

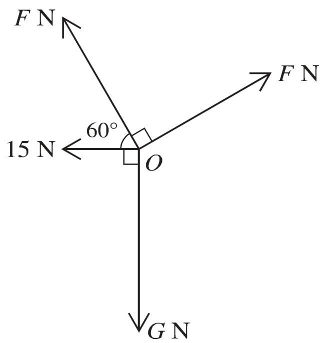
(ii) Resolving the forces vertically,

$$\sum F_y = 25\sin\alpha + 30\sin\beta - 20$$
$$= (25)(0.28) + (30)(0.6) - 20 = 7 + 18 - 20 = 5\text{ N}$$

$\therefore$ the resultant is of magnitude 5 N acting vertically along the y-axis. **(Ans).**

(iii) Magnitude of the replacement force is 30 N with direction vertically downward along y-axis.

Question 7



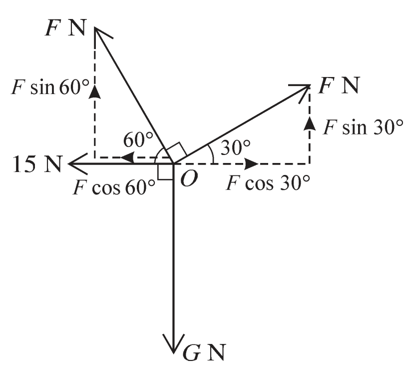
Four horizontal forces act at a point O and are in equilibrium. The magnitudes of the forces are F N, G N, 15 N and F N, and the forces act in directions as shown in the diagram.

(i) Show that $F = 41.0$, correct to 3 significant figures. [3]

(ii) Find the value of G. [2]

[N15/P4/Q1]

Suggested Solution:



(i) Resolving the forces horizontally, we have,

$\rightarrow$: $F\cos 30° = 15 + F\cos 60°$

$\Rightarrow$ $F\cos 30° - F\cos 60° = 15$

$F(\cos 30° - \cos 60°) = 15$

$0.366F = 15$

$F = 40.983 \approx 41.0$ N **(Shown).**

(ii) Resolving the forces vertically,

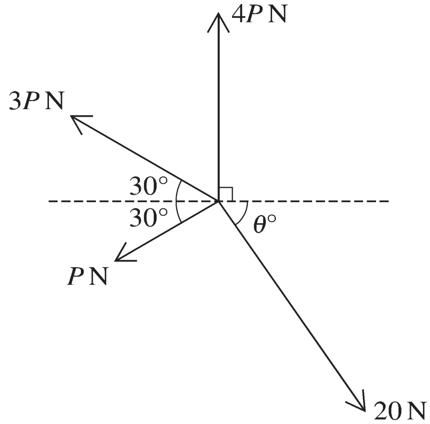
$\uparrow$: $F\sin 30° + F\sin 60° = G$

$F(\sin 30° + \sin 60°) = G$

$(40.983)(0.5 + 0.866) = G$

$G = 55.983 \approx 56.0$ N **(Ans).**

Question 15

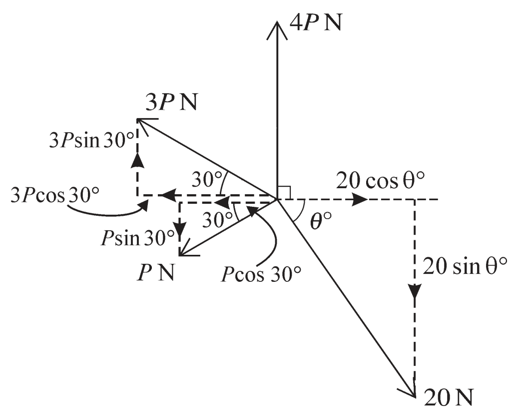


Coplanar forces of magnitudes 20 N, P N, $3P$ N and $4P$ N act at a point in the directions shown in the diagram. The system is in equilibrium.

Find P and θ. [6]

[J20/P4/Q2]

Suggested Solution:



Resolving the forces horizontally, we have,

$\rightarrow$: $20\cos\theta° = 3P\cos 30° + P\cos 30°$

$20\cos\theta° = 4P\cos 30°$ .... (1)

Resolving the forces vertically,

$\uparrow$: $4P + 3P\sin 30° = P\sin 30° + 20\sin\theta°$

$4P + 2P\sin 30° = 20\sin\theta°$

$20\sin\theta° = 5P$ .... (2)

Divide eq. (2) by eq. (1),

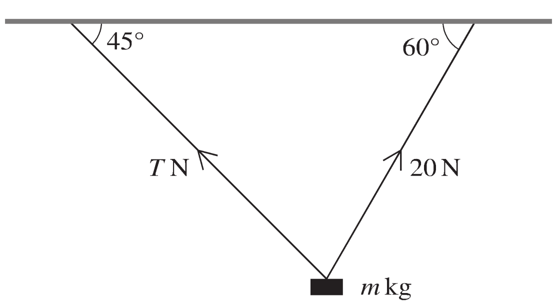
$$\frac{20\sin\theta°}{20\cos\theta°} = \frac{5P}{4P\cos 30°}$$

$\Rightarrow \tan\theta = \dfrac{5}{3.464} \Rightarrow \tan\theta = 1.443 \Rightarrow \theta = 55.3°$ **(Ans).**

Substitute $\theta = 55.3°$ into (2),

$5P = 20\sin 55.3° \Rightarrow P = 3.29$ N **(Ans).**

Question 16

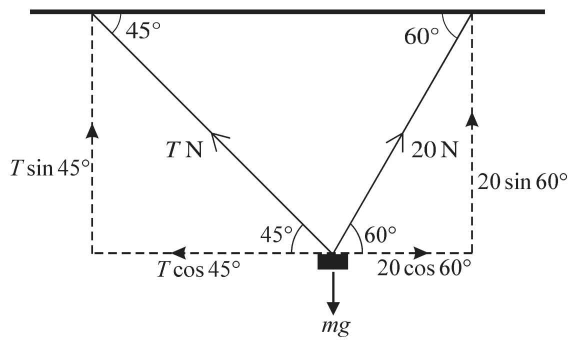


A block of mass m kg is held in equilibrium below a horizontal ceiling by two strings, as shown in the diagram. One of the strings is inclined at 45° to the horizontal and the tension in this string is T N. The other string is inclined at 60° to the horizontal and the tension in this string is 20 N.

Find T and m. [5]

[N20/P4/Q3]

Suggested Solution:



Resolving the forces horizontally,

$\rightarrow$: $T\cos 45° = 20\cos 60°$

$\Rightarrow \quad T = \dfrac{20\cos 60°}{\cos 45°} = 14.1 \text{ N}$ **(Ans).**

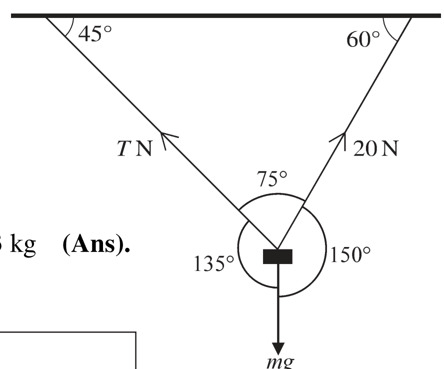
Resolving the forces vertically

$\uparrow$: $T\sin 45° + 20\sin 60° = mg$

$\Rightarrow \quad (14.14)(0.707) + 20(0.866) = 10m$

$\Rightarrow \quad 27.32 = 10m \quad \Rightarrow \quad m = 2.73$ kg **(Ans).**

Alternative Solution:



Using Lami's theorem,

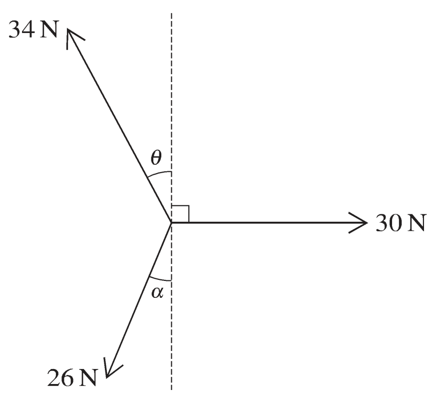
$$\frac{T}{\sin 150°} = \frac{20}{\sin 135°}$$

$$T = \frac{20}{\sin 135°} \times \sin 150° = 14.14 \text{ N} \quad \textbf{(Ans).}$$

Also, $\dfrac{mg}{\sin 75°} = \dfrac{20}{\sin 135°}$

$$10m = \frac{20}{\sin 135°} \times \sin 75° \quad \Rightarrow \quad 10m = 27.32 \quad \Rightarrow \quad m = 2.73 \text{ kg} \quad \textbf{(Ans).}$$

Question 17



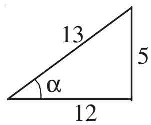
Coplanar forces of magnitudes 34 N, 30 N and 26 N act at a point in the directions shown in the diagram.

Given that $\sin\alpha = \dfrac{5}{13}$ and $\sin\theta = \dfrac{8}{17}$, find the magnitude and direction of the resultant of the three forces. [6]

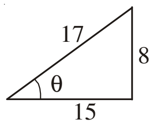
[J21/P4/Q2]

Suggested Solution:

Given that, $\sin\alpha = \dfrac{5}{13} \;\Rightarrow\; \cos\alpha = \dfrac{12}{13}$



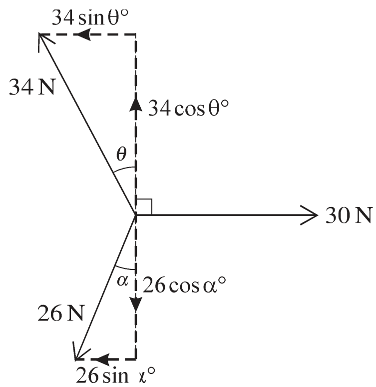
$\sin\theta = \dfrac{8}{17} \;\Rightarrow\; \cos\theta = \dfrac{15}{17}$



Resolving the forces horizontally and vertically,

$$\sum F_x = 30 - 34\sin\theta - 26\sin\alpha$$
$$= 30 - 34\left(\frac{8}{17}\right) - 26\left(\frac{5}{13}\right) = 30 - 16 - 10 = 4$$

$$\sum F_y = 34\cos\theta - 26\cos\alpha$$
$$= 34\left(\frac{15}{17}\right) - 26\left(\frac{12}{13}\right) = 30 - 24 = 6$$

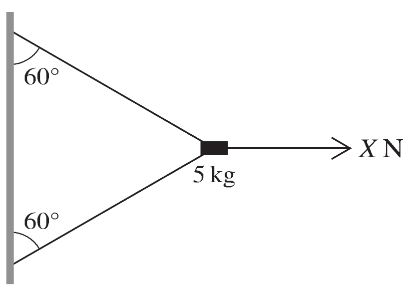


$\therefore$ Resultant force, $R = \sqrt{\left(\sum F_x\right)^2 + \left(\sum F_y\right)^2}$

$= \sqrt{4^2 + 6^2} = \sqrt{52} = 2\sqrt{13}$ N **(Ans).**

Direction, $\theta = \tan^{-1}\left(\dfrac{6}{4}\right) = 56.3°$ anticlockwise from 30 N force. **(Ans).**

Question 18



A block of mass 5 kg is held in equilibrium near a vertical wall by two light strings and a horizontal force of magnitude X N, as shown in the diagram. The two strings are both inclined at 60° to the vertical.

(a) Given that $X = 100$, find the tension in the lower string. [4]

(b) Find the least value of X for which the block remains in equilibrium in the position shown. [4]

[N21/P4/Q6]

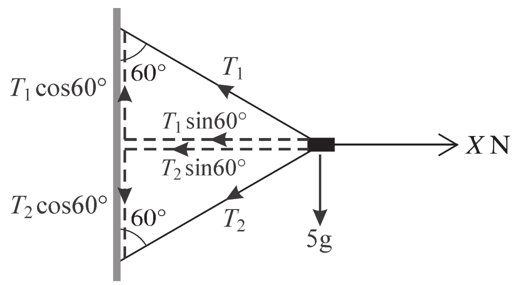
Learning CORNER

Suggested Solution:

(a) Resolving the forces horizontally,

$\rightarrow$: $X = T_1 \sin 60° + T_2 \sin 60°$

$\Rightarrow \quad 100 = (T_1 + T_2)\sin 60° \quad \Rightarrow \quad T_1 + T_2 = 115.47$ .... (1)



Resolving the forces vertically,

$\uparrow$: $T_1 \cos 60° = T_2 \cos 60° + 5g$

$\Rightarrow \quad T_1 \cos 60° - T_2 \cos 60° = 50$

$\Rightarrow \quad (T_1 - T_2)\cos 60° = 50 \quad \Rightarrow \quad T_1 - T_2 = 100$ .... (2)

Solving equations (1) & (2) simultaneously,

$$\begin{aligned} T_1 + T_2 &= 115.47 \\ T_1 - T_2 &= 100 \\ \underline{-\quad +\quad -} & \\ 2T_2 = 15.47 \quad \Rightarrow \quad T_2 &= 7.735 \approx 7.74 \end{aligned}$$

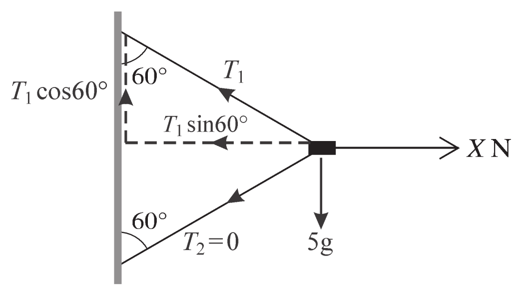
$\therefore$ Tension in the lower string = 7.74 N **(Ans).**

(b) When X has least value, the lower string becomes slack

$\therefore$ Tension in $T_2 = 0$

Resolving the forces horizontally and vertically,

$\rightarrow$: $X = T_1 \sin 60°$ .... (1)



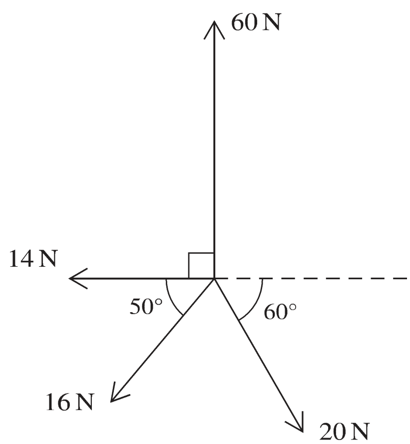
$\uparrow$: $T_1 \cos 60° = 5g$

$\Rightarrow \quad T_1(0.5) = 50 \quad \Rightarrow \quad T_1 = 100$

Substitute $T_1 = 100$ into (1), $X = 100\sin 60° = 86.6$

$\therefore$ Least value of $X = 86.6$ N. **(Ans).**

Question 19

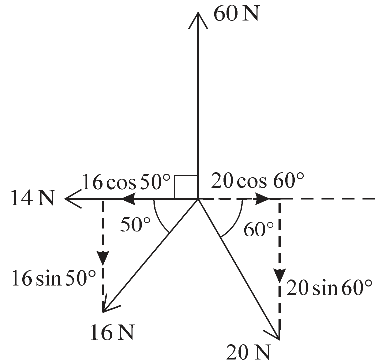


Coplanar forces of magnitudes 60 N, 20 N, 16 N and 14 N act at a point in the directions shown in the diagram.

Find the magnitude and direction of the resultant force. [6]

[J22/P4/Q2]

Suggested Solution:



Resolving forces horizontally and vertically, we have,

$\sum F_x = 20\cos 60° - 16\cos 50° - 14 = -14.285$ N

$\sum F_y = 60 - 16\sin 50° - 20\sin 60° = 30.423$ N

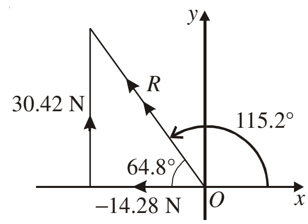
$\therefore$ Resultant force, $R = \sqrt{F_x{}^2 + F_y{}^2}$

$= \sqrt{(-14.285)^2 + (30.423)^2}$

$= \sqrt{1129.620} = 33.61$ N **(Ans).**

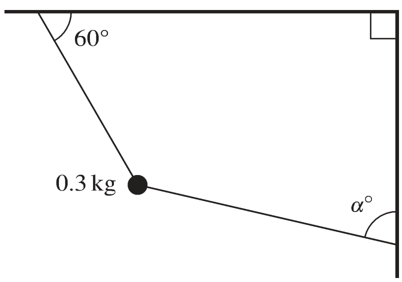
Let θ be the angle which the resultant force makes with the x-axis

$$\theta = \tan^{-1}\left(\frac{Fy}{Fx}\right) = \tan^{-1}\left(\frac{30.423}{14.285}\right) = 64.8°$$



$\therefore$ Direction of the resultant force is $180° - 64.8° = 115.2°$ anticlockwise from the positive x-axis. **(Ans).**

Question 20

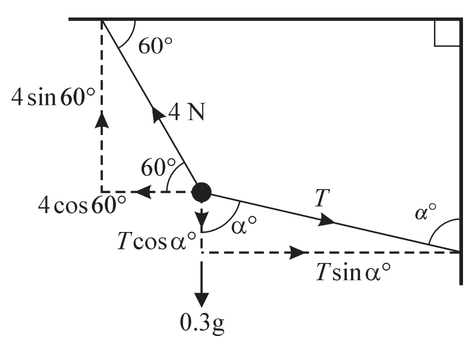


A particle of mass 0.3 kg is held at rest by two light inextensible strings. One string is attached at an angle of 60° to a horizontal ceiling. The other string is attached at an angle $\alpha°$ to a vertical wall (see diagram). The tension in the string attached to the ceiling is 4 N.

Find the tension in the string which is attached to the wall and find the value of α. [6]

[N22/P4/Q3]

Suggested Solution:



Resolving the forces horizontally,

$\rightarrow$: $T\sin\alpha° = 4\cos 60° \Rightarrow T\sin\alpha° = 2$ .... (1)

Resolving the forces vertically,

$\uparrow$: $4\sin 60° = 0.3g + T\cos\alpha°$

$\Rightarrow 3.4641 = 3 + T\cos\alpha° \Rightarrow T\cos\alpha° = 0.4641$ .... (2)

Squaring and adding eq. (1) and eq. (2)

$T^2\sin^2\alpha° + T^2\cos^2\alpha° = 2^2 + (0.4641)^2$

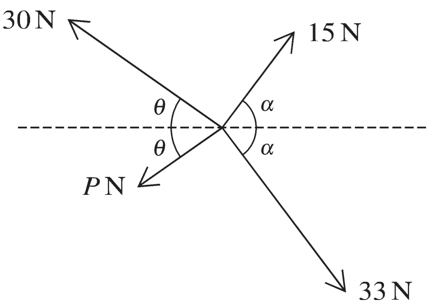
$\Rightarrow T^2(\sin^2\alpha° + \cos^2\alpha°) = 4.2154$

$\Rightarrow T^2 = 4.2154 \Rightarrow T = 2.053 \approx 2.05$ N **(Ans).**

Substitute $T = 2.053$ into eq. (1),

$(2.053)\sin\alpha° = 2 \Rightarrow \sin\alpha° = 0.9742 \Rightarrow \alpha° = 76.9°$ **(Ans).**

Question 21



Coplanar forces of magnitudes 30 N, 15 N, 33 N and P N act at a point in the directions shown in the diagram, where $\tan\alpha = \frac{4}{3}$. The system is in equilibrium.

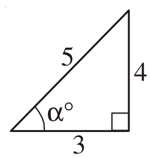
(a) Show that $\left(\frac{14.4}{30-P}\right)^2 + \left(\frac{28.8}{P+30}\right)^2 = 1$. [4]

(b) Verify that $P = 6$ satisfies this equation and find the value of θ. [2]

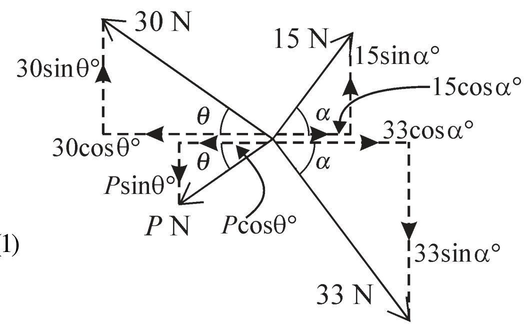
[J23/P4/Q3]

Suggested Solution:

(a) Given that, $\tan\alpha = \frac{4}{3}$



$\Rightarrow \quad \sin\alpha = \frac{4}{5} \quad$ and $\quad \cos\alpha = \frac{3}{5}$



Resolving the forces horizontally, we have,

$\rightarrow$: $\quad 15\cos\alpha° + 33\cos\alpha° = 30\cos\theta° + P\cos\theta°$

$\Rightarrow \quad 48\cos\alpha° = 30\cos\theta° + P\cos\theta°$

$\Rightarrow \quad 48\left(\frac{3}{5}\right) = (30+P)\cos\theta°$

$\Rightarrow \quad 28.8 = (30+P)\cos\theta° \quad \Rightarrow \quad \cos\theta° = \frac{28.8}{30+P}$ .... (1)

Resolving the forces vertically,

$\uparrow$: $\quad 30\sin\theta° + 15\sin\alpha° = P\sin\theta° + 33\sin\alpha°$

$\Rightarrow \quad 30\sin\theta° - P\sin\theta° = 33\sin\alpha° - 15\sin\alpha°$

$\Rightarrow \quad 30\sin\theta° - P\sin\theta° = 18\sin\alpha°$

$\Rightarrow \quad (30-P)\sin\theta° = 18\left(\frac{4}{5}\right)$

$\Rightarrow \quad (30-P)\sin\theta° = 14.4 \quad \Rightarrow \quad \sin\theta° = \frac{14.4}{30-P}$ .... (2)

Squaring and adding equations (1) & (2),

$$\sin^2\theta° + \cos^2\theta° = \left(\frac{14.4}{30-P}\right)^2 + \left(\frac{28.8}{30+P}\right)^2$$

since, $\sin^2\theta° + \cos^2\theta° = 1$

$\Rightarrow \quad \left(\frac{14.4}{30-P}\right)^2 + \left(\frac{28.8}{30+P}\right)^2 = 1$ **(Shown).**

(b) $\left(\dfrac{14.4}{30-P}\right)^2+\left(\dfrac{28.8}{P+30}\right)^2=1$

When $P=6$,

$\Rightarrow\ \left(\dfrac{14.4}{30-6}\right)^2+\left(\dfrac{28.8}{6+30}\right)^2=1$

$\Rightarrow\ \left(\dfrac{14.4}{24}\right)^2+\left(\dfrac{28.8}{36}\right)^2=1$

$\Rightarrow\ (0.6)^2+(0.8)^2=1$

$\Rightarrow\ 0.36+0.64=1$

$\Rightarrow\ 1=1$

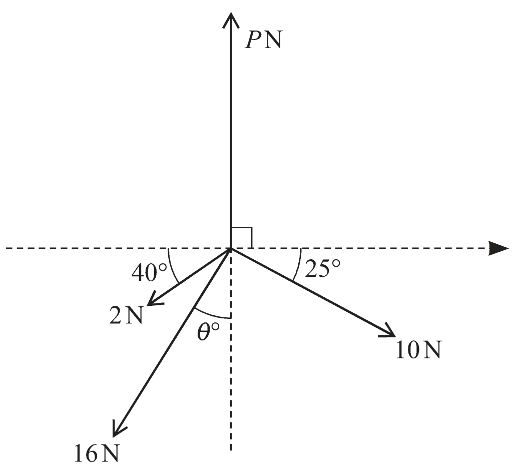
$\therefore\ P=6$ satisfies the equation.

Now, using equation (2) of part **(a)**, $\sin\theta°=\dfrac{14.4}{30-P}$

$\Rightarrow\ \sin\theta°=\dfrac{14.4}{30-6}$

$\Rightarrow\ \sin\theta°=0.6 \ \Rightarrow\ \theta°=\sin^{-1}(0.6)=36.9°$ **(Ans).**

Question 22



Four coplanar forces of magnitude P N, 10 N, 16 N and 2 N act at a point in the directions shown in the diagram. It is given that the forces are in equilibrium.

Find the values of θ and P. [6]

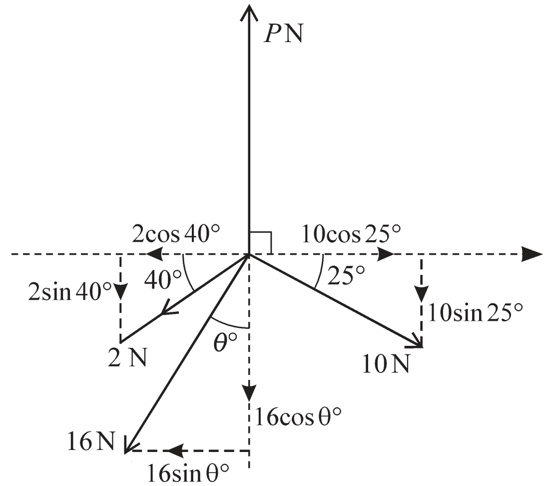
[J24/P4/Q3]

Suggested Solution:

Resolving the forces horizontally, we have,

$\rightarrow$: $2\cos 40° + 16\sin\theta° = 10\cos 25°$

$$16\sin\theta° = 10\cos 25° - 2\cos 40°$$
$$16\sin\theta° = 7.531$$
$$\sin\theta° = 0.4707$$
$$\theta° = 28.1°$$



Resolving the forces vertically,

$\uparrow$: $P = 2\sin 40° + 10\sin 25° + 16\cos\theta°$

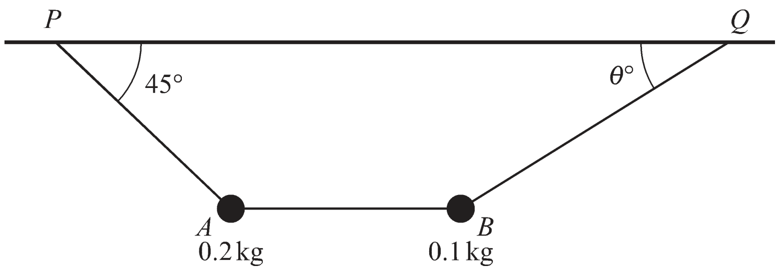
$P = 2\sin 40° + 10\sin 25° + 16\cos 28.1°$

$P = 1.2856 + 4.2262 + 14.1140$

$P = 19.63$ N

$\therefore$ $\theta = 28.1°$, $P = 19.6$ N **(Ans).**

Question 23



The diagram shows two particles, A and B, of masses 0.2 kg and 0.1 kg respectively. The particles are suspended below a horizontal ceiling by two strings, AP and BQ, attached to fixed points P and Q on the ceiling. The particles are connected by a horizontal string, AB. Angle $APQ = 45°$ and $BQP = \theta°$. Each string is light and inextensible. The particles are in equilibrium.

(a) Find the value of the tension in the string AB. [2]

(b) Find the value of θ and the tension in the string BQ. [4]

[N24/P4/Q4]

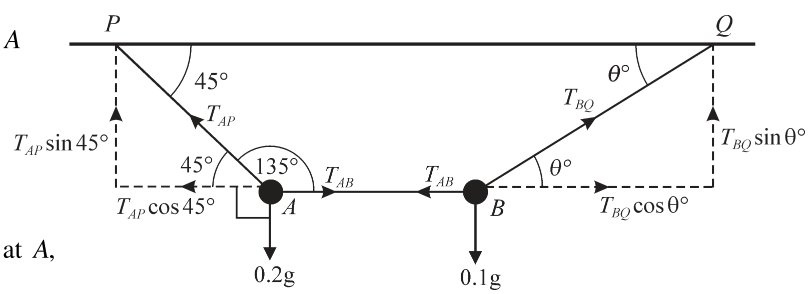
Suggested Solution:

(a) Resolving the forces vertically at A

$\uparrow$: $T_{AP}\sin 45° = 0.2g$

$\Rightarrow$ $T_{AP}\left(\dfrac{1}{\sqrt{2}}\right) = 2$

$\Rightarrow$ $T_{AP} = 2\sqrt{2}$



Resolving the forces horizontally at A,

$\rightarrow$: $T_{AB} = T_{AP}\cos 45°$

$\Rightarrow$ $T_{AB} = 2\sqrt{2}\left(\dfrac{1}{\sqrt{2}}\right)$ $\Rightarrow$ $T_{AB} = 2$

$\therefore$ tension in the string $AB = 2$ N **(Ans).**

Alternative Solution:

Using Lami's theorem,

$$\frac{T_{AB}}{\sin(45^\circ + 90^\circ)} = \frac{0.2g}{\sin 135^\circ}$$

$$\frac{T_{AB}}{\sin 135^\circ} = \frac{0.2g}{\sin 135^\circ}$$

$$T_{AB} = 0.2g$$

$$T_{AB} = 0.2(10) = 2 \text{ N} \quad \textbf{(Ans).}$$

(b) Resolving the forces horizontally and vertically at B

$\uparrow$: $T_{BQ} \sin\theta^\circ = 0.1g$ .... (1)

$\rightarrow$: $T_{BQ} \cos\theta^\circ = T_{AB}$ .... (2)

Divide (1) by (2)

$$\frac{T_{BQ}\sin\theta^\circ}{T_{BQ}\cos\theta^\circ} = \frac{0.1g}{T_{AB}}$$

$$\Rightarrow \quad \frac{\sin\theta^\circ}{\cos\theta^\circ} = \frac{1}{2}$$

$$\Rightarrow \quad \tan\theta^\circ = \frac{1}{2} \quad \Rightarrow \quad \theta^\circ = 26.565^\circ \approx 26.6^\circ \quad \textbf{(Ans).}$$

Substitute value of θ into (1),

$$T_{BQ} \sin 26.565^\circ = 0.1g$$

$$\Rightarrow \quad T_{BQ} = \frac{0.1(10)}{\sin 26.565^\circ} \quad \Rightarrow \quad T_{BQ} = \frac{1}{0.4472} \quad \Rightarrow \quad T_{BQ} = 2.24 \text{ N}$$

$\therefore$ Tension in string $BQ = 2.24$ N **(Ans).**

Topic 6

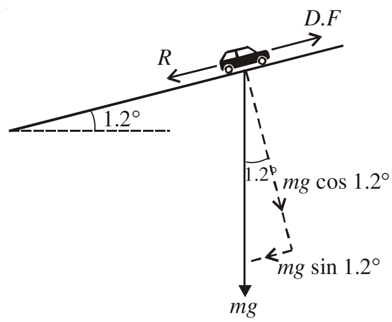
Energy, Work and Power

Question 1

A car of mass 1150 kg travels up a straight hill inclined at 1.2° to the horizontal. The resistance to motion of the car is 975 N. Find the acceleration of the car at an instant when it is moving with speed 16 ms^{-1} and the engine is working at a power of 35 kW. [4]

[J10/P4/Q1]

Suggested Solution:



Driving force, $D.F = \dfrac{P}{v} = \dfrac{35000}{16} = 2187.5\text{ N}$

Applying Newton's 2nd law,

Net force $= ma$

$\Rightarrow \quad D.F - R - mg\sin 1.2° = ma$

$\Rightarrow \quad 2187.5 - 975 - (1150)(10)\sin 1.2° = 1150a$

$971.662 = 1150a \quad \Rightarrow \quad a = 0.845\text{ m/s}^2$ **(Ans).**

Question 2

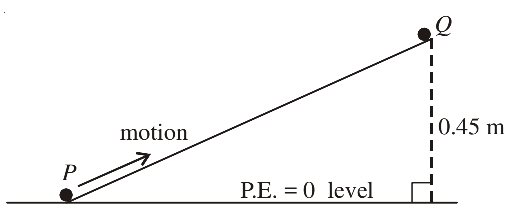
P and *Q* are fixed points on a line of greatest slope of an inclined plane. The point *Q* is at a height of 0.45 m above the level of *P*. A particle of mass 0.3 kg moves upwards along the line *PQ*.

(i) Given that the plane is smooth and that the particle just reaches *Q*, find the speed with which it passes through *P*. [3]

(ii) It is given instead that the plane is rough. The particle passes through *P* with the same speed as that found in part **(i)**, and just reaches a point *R* which is between *P* and *Q*. The work done against the frictional force in moving from *P* to *R* is 0.39 J. Find the potential energy gained by the particle in moving from *P* to *R* and hence find the height of *R* above the level of *P*. [4]

[J10/P4/Q5]

Suggested Solution:



(i) Let v ms^{-1} be the speed of the particle at *P*.

Given that the plane is smooth,

$\therefore$ K.E. at *P* = P.E. at *Q*

$$\frac{1}{2}mv^2 = mgh$$

$$\frac{1}{2}v^2 = (10)(0.45)$$

$v^2 = 9 \quad \Rightarrow \quad v = 3\text{ ms}^{-1}$ **(Ans).**

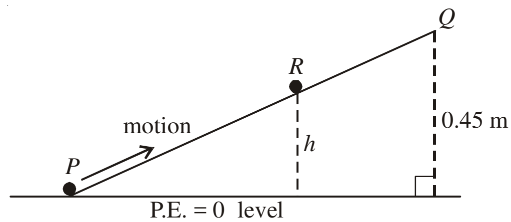
Learning CORNER

(ii) Applying work-energy relation,

gain in P.E at R = loss in K.E at P – work done against resistance

$$= \frac{1}{2}(0.3)(3)^2 - 0.39$$

$$= 1.35 - 0.39 = 0.96 \text{ J} \quad \textbf{(Ans).}$$



Let h be the height of point R.

P.E. at $R = 0.96$

$\Rightarrow \quad (0.3)(10)(h) = 0.96$

$3h = 0.96 \quad \Rightarrow \quad h = 0.32$ m **(Ans).**

Question 3

A cyclist, working at a constant rate of 400 W, travels along a straight road which is inclined at 2° to the horizontal. The total mass of the cyclist and his cycle is 80 kg. Ignoring any resistance to motion, find, correct to 1 decimal place, the acceleration of the cyclist when he is travelling

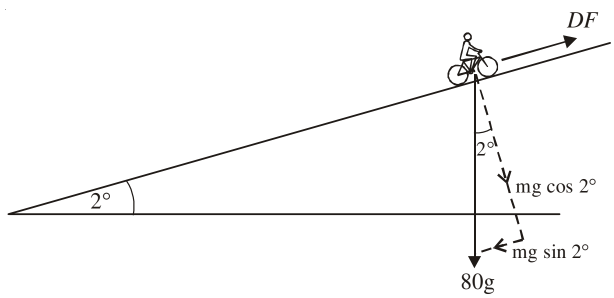
(i) uphill at 4 ms^{-1},

(ii) downhill at 4 ms^{-1}. [5]

[N10/P4/Q2]

Suggested Solution:

(i) Driving force, $DF = \dfrac{\text{power}}{\text{velocity}} = \dfrac{400}{4} = 100 \text{ N}$



applying newton's 2nd law

Net force = ma

$\Rightarrow \quad DF - mg\sin 2° = ma$

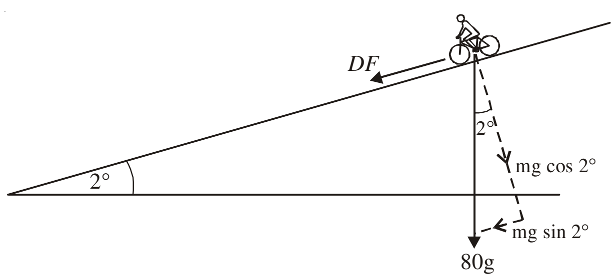
$100 - 800\sin 2° = 80a$

$$a = \frac{100 - 800\sin 2°}{80}$$

$= 0.901$ ms^{-2} **(Ans).**

(ii) Driving force, $DF = \dfrac{\text{power}}{\text{velocity}}$

$$= \frac{400}{4} = 100 \text{ N}$$



applying newton's 2nd law

Net force = ma

$\Rightarrow \quad DF + mg\sin 2° = ma$

$100 + 800\sin 2° = 80a$

$$a = \frac{100 + 800\sin 2°}{80}$$

$= 1.599 \approx 1.60$ ms^{-2} **(Ans).**

Question 4

A block of mass 20 kg is pulled from the bottom to the top of a slope. The slope has length 10 m and is inclined at 4.5° to the horizontal. The speed of the block is 2.5 ms^{-1} at the bottom of the slope and 1.5 ms^{-1} at the top of the slope.

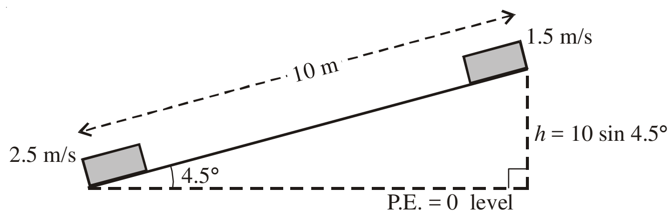
(i) Find the loss of kinetic energy and the gain in potential energy of the block. [3]

(ii) Given that the work done against the resistance to motion is 50 J, find the work done by the pulling force acting on the block. [2]

(iii) Given also that the pulling force is constant and acts at an angle of 15° upwards from the slope, find its magnitude. [2]

[N10/P4/Q4]

Suggested Solution:



(i) K.E. at bottom $= \frac{1}{2}(20)(2.5)^2 = 62.5$ J

K.E. at the top $= \frac{1}{2}(20)(1.5)^2 = 22.5$ J

$\therefore$ loss in K.E. $= 62.5 - 22.5 = 40$ J **(Ans).**

P.E. at bottom $= (20)(10)(0) = 0$ J

P.E. at the top $= (20)(10)(10\sin 4.5°) = 156.92$ J

$\therefore$ gain in P.E. $= 156.9 - 0 = 156.92 \approx 157$ J **(Ans).**

(ii) Total mechanical energy at the bottom = K.E. + P.E.

$= 62.5 + 0 = 62.5$ J

Total mechanical energy at the top = K.E. + P.E.

$= 22.5 + 156.92 = 179.42$ J

using work-energy relation,

Work done by net force = Change in mechanical energy

$\Rightarrow$ W.D by pulling force − W.D against resistance = change in M.E.

$\Rightarrow$ W.D by pulling force $- 50 = 179.42 - 62.5$

$\Rightarrow$ W.D by pulling force $= 116.92 + 50 = 166.92 \approx 167$ J **(Ans).**

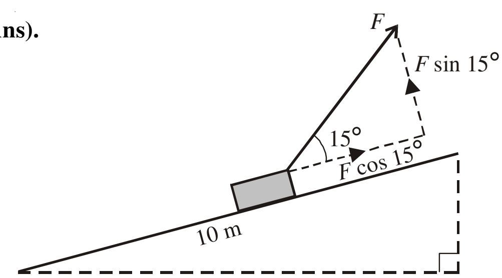
(iii) Using, work done $= F \times d$

W.D. by pulling force $= (F\cos 15°)(10)$

$$166.92 = 10F\cos 15°$$

$$F = \frac{166.92}{10\cos 15°} = 17.28 \approx 17.3$$

$\therefore$ magnitude of pulling force = 17.3 N **(Ans).**



Question 5

A load is pulled along horizontal ground for a distance of 76 m, using a rope. The rope is inclined at 5° above the horizontal and the tension in the rope is 65 N.

(i) Find the work done by the tension. [2]

At an instant during the motion the velocity of the load is 1.5 ms^{-1}.

(ii) Find the rate of working of the tension at this instant. [2]

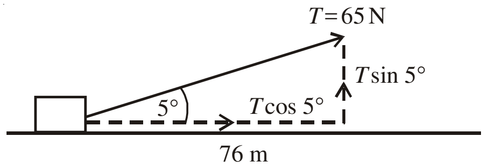
[J11/P4/Q1]

Suggested Solution:

(i) Work Done $= (T\cos\theta)\times$ distance

$= 65\cos 5°\times 76$

$= 4921.2$ J $= 4.92$ kJ **(Ans).**



(ii) Rate of doing work = Power

= force × velocity

$= (65\cos 5°)(1.5) = 97.1$ W.

> ***Question 6***
>
> An object of mass 8 kg slides down a line of greatest slope of an inclined plane. Its initial speed at the top of the plane is 3 ms^{-1} and its speed at the bottom of the plane is 8 ms^{-1}. The work done against the resistance to motion of the object is 120 J. Find the height of the top of the plane above the level of the bottom. [4]
>
> *[J11/P4/Q2]*

Suggested Solution:

K.E. at the top $= \frac{1}{2}(8)(3)^2 = 36$ J, K.E. at bottom $= \frac{1}{2}(8)(8)^2 = 256$ J

$\therefore$ gain in K.E. $= 256 - 36 = 220$ J

P.E. at the top $= (8)(10)h = (80h)$ J, P.E. at bottom = 0 J

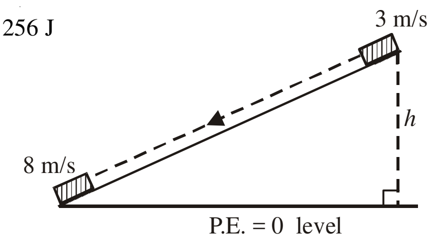
$\therefore$ loss in P.E. $= 80h$ J

using work-energy relation,

Loss in P.E = gain in K.E + work done against resistance

$80h = 220 + 120$

$h = 4.25$ m **(Ans).**



> ***Question 7***
>
> A racing cyclist, whose mass with his cycle is 75 kg, works at a rate of 720 W while moving on a straight horizontal road. The resistance to the cyclist's motion is constant and equal to R N.
>
> **(i)** Given that the cyclist is accelerating at 0.16 ms^{-2} at an instant when his speed is 12 ms^{-1}, find the value of R. [3]
>
> **(ii)** Given that the cyclist's acceleration is positive, show that his speed is less than 15 ms^{-1}. [2]
>
> *[N11/P4/Q1]*

Suggested Solution:

(i) We have, $m = 75$ kg, $a = 0.16$ ms^{-2}, $v = 12$ ms^{-1}, $P = 720$ W

Driving force, $DF = \frac{P}{v} = \frac{720}{12} = 60$ N

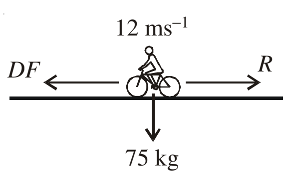
applying Newtons 2nd law,

Net Force $= ma$

$\Rightarrow$ $DF - R = ma$

$60 - R = (75)(0.16)$

$R = 60 - 12 = 48$ N **(Ans).**



(ii) Let maximum velocity of the cyclist be v_{max}

at maximum velocity, $a = 0$,

$\Rightarrow \quad DF = R$

$\Rightarrow \quad \dfrac{720}{v_{max}} = 48 \quad \Rightarrow \quad v_{max} = \dfrac{720}{48} = 15 \text{ ms}^{-1}$

as the cyclist is accelerating, it did not reach its maximum velocity.

$\Rightarrow$ speed of the cyclist is less than 15 ms^{-1} **(Shown).**

Question 8

A lorry of mass 16 000 kg climbs a straight hill *ABCD* which makes an angle θ with the horizontal, where $\sin\theta = \frac{1}{20}$. For the motion from *A* to *B*, the work done by the driving force of the lorry is 1200 kJ and the resistance to motion is constant and equal to 1240 N. The speed of the lorry is 15 ms^{-1} at *A* and 12 ms^{-1} at *B*.

(i) Find the distance *AB*. [5]

For the motion from *B* to *D* the gain in potential energy of the lorry is 2400 kJ.

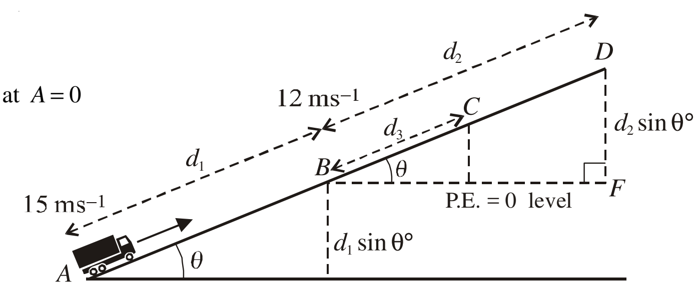
(ii) Find the distance *BD*. [1]

For the motion from *B* to *D* the driving force of the lorry is constant and equal to 7200 N. From *B* to *C* the resistance to motion is constant and equal to 1240 N and from *C* to *D* the resistance to motion is constant and equal to 1860 N.

(iii) Given that the speed of the lorry at *D* is 7 ms^{-1}, find the distance *BC*. [4]

[N11/P4/Q6]

Suggested Solution:



(i) Let the distance *AB* be d_1.

K.E at $A = \frac{1}{2}(16000)(15)^2 = 1800000$ J, P.E at $A = 0$

$\therefore$ Total mechanical energy at $A = 1800000$ J

K.E at $B = \frac{1}{2}(16000)(12)^2 = 1152000$ J

P.E at $B = (16000)(10)(d_1 \sin\theta)$

$= (160000)(d_1)(\frac{1}{20}) = 8000 d_1$

$\therefore$ Total mechanical energy at $B = 1152000 + 8000 d_1$

Using work-energy relation,

work done by net force = change in mechanical energy

$\Rightarrow$ W.D by driving force − W.D against resistance = M.E at B − M.E at A

$\Rightarrow \quad 1200000 - 1240 \times d_1 = (1152000 + 8000 d_1) - 1800000$

$\Rightarrow \quad 8000 d_1 + 1240 d_1 = 1200000 - 1152000 + 1800000$

$\Rightarrow \quad 9240 d_1 = 1848000 \quad \Rightarrow \quad d_1 = 200$

$\therefore$ distance $AB = 200$ m **(Ans).**

(ii) Let the distance from B to D be d_2.

gain in P.E. from B to $D = mg(d_2 \sin\theta)$

$\Rightarrow \quad 2400000 = (16000)(10)(d_2 \times \frac{1}{20})$

$\Rightarrow \quad 2400000 = 8000 d_2 \quad \Rightarrow \quad d_2 = 300$ m

$\therefore$ distance $BD = 300$ m **(Ans).**

(iii) Let the distance BC be d_3.

consider the line BF as $\text{P.E.} = 0$ level

Total M.E at $D = \text{K.E.} + \text{P.E.}$

$$= \frac{1}{2}(16000)(7)^2 + 2400000 = 2792000 \text{ J}$$

Total M.E at $B = \text{K.E.} + \text{P.E.}$

$$= \frac{1}{2}(16000)(12)^2 + 0 = 1152000 \text{ J}$$

Using work-energy relation,

W.D by driving force − W.D against resistance = M.E at D − M.E at B

$\Rightarrow \quad (7200)(300) - [(1240)(d_3) + (1860)(300 - d_3)] = 2792000 - 1152000$

$\Rightarrow \quad 2160000 - (1240d_3 + 558000 - 1860d_3) = 1640000$

$\Rightarrow \quad 2160000 - 558000 + 620d_3 = 1640000$

$\Rightarrow \quad 620d_3 = 38000$

$\Rightarrow \quad d_3 = 61.3 \text{ m}$

$\therefore \quad$ distance $BC = 61.3$ m **(Ans).**

Question 9

A block is pulled in a straight line along horizontal ground by a force of constant magnitude acting at an angle of 60° above the horizontal. The work done by the force in moving the block a distance of 5 m is 75 J. Find the magnitude of the force. [3]

[J12/P4/Q1]

Suggested Solution:

(i) Given that,

Work done = 75 J, distance = 5 m

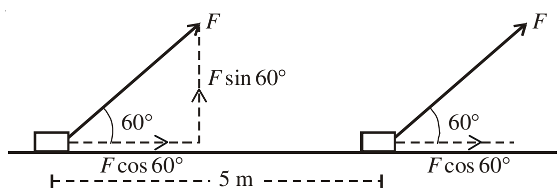
work done = force × distance

$\Rightarrow \quad 75 = F\cos 60° \times 5$

$\Rightarrow \quad 75 = F(0.5) \times 5$

$\Rightarrow \quad 75 = 2.5F$

$\Rightarrow \quad F = 30 \text{ N}$ **(Ans).**



Question 10

A car of mass 1250 kg travels from the bottom to the top of a straight hill which has length 400 m and is inclined to the horizontal at an angle of α, where $\sin\alpha = 0.125$. The resistance to the car's motion is 800 N. Find the work done by the car's engine in each of the following cases.

(i) The car's speed is constant. [4]

(ii) The car's initial speed is 6 m s^{-1}, the car's driving force is 3 times greater at the top of the hill than it is at the bottom, and the car's power output is 5 times greater at the top of the hill than it is at the bottom. [5]

[J12/P4/Q6]

Learning CORNER

Suggested Solution:

(i) Given that, $m = 1250$ kg, $\sin\alpha = 0.125$, resistance, $R = 800$ N,

Using work-energy relation,

work done by net force = change in P.E.

$\Rightarrow$ W.D by driving force – W.D against resistance = P.E. at top – P.E. at bottom

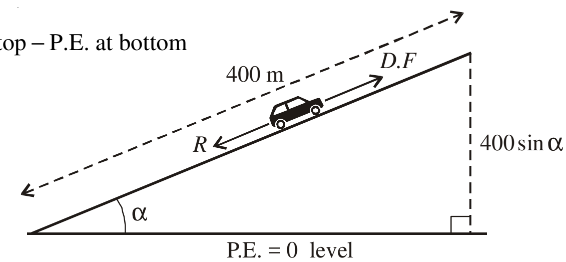
$\Rightarrow$ W.D by car's engine $-(800)(400) = mg(400\sin\alpha) - 0$

$\Rightarrow$ W.D by car's engine $- 320000 = (1250)(10)(400\times 0.125)$

$\Rightarrow$ W.D by car's engine $= 625000 + 320000$

$= 945000$ J

$= 945$ kJ **(Ans).**



(ii) Let the driving force be F_D and power output be P.

At the bottom of the hill, using $P = F\times v$

driving force, $F_D = \dfrac{P}{6}$

At the top of the hill, driving force $= 3F_D$, power $= 5P$

$\therefore \quad 3F_D = \dfrac{5P}{v}$

$\Rightarrow \quad 3(\dfrac{P}{6}) = \dfrac{5P}{v} \quad \Rightarrow \quad \dfrac{1}{2} = \dfrac{5}{v} \quad \Rightarrow \quad v = 10$

$\therefore$ velocity of the car at the top of hill $= 10$ ms^{-1}

Also,

Total mechanical energy at the bottom = K.E. + P.E

$= \dfrac{1}{2}(1250)(6)^2 + 0$

$= 22500$ J

Total mechanical energy at the top = K.E. + P.E

$= \dfrac{1}{2}(1250)(10)^2 + (1250)(10)(400\times 0.125)$

$= 687500$ J

Using work-energy relation,

work done by net force = change in mechanical energy

$\Rightarrow$ W.D by driving force – W.D against resistance = M.E. at top – M.E. at bottom

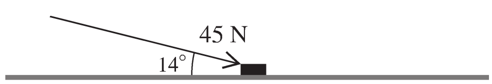
$\Rightarrow$ W.D by car's engine $-(800)(400) = 687500 - 22500$

$\Rightarrow$ W.D by car's engine $- 320000 = 665000$

$\Rightarrow$ W.D by car's engine $= 985000$ J

$= 985$ kJ **(Ans).**

Question 11

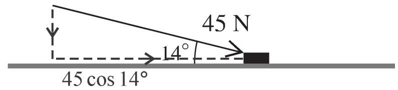


A block is pushed along a horizontal floor by a force of magnitude 45 N acting at an angle of 14° to the horizontal (see diagram). Find the work done by the force in moving the block a distance of 25 m. [3]

[N12/P4/Q1]

Suggested Solution:

Given that, Force = 45 N, distance = 25 m



$$\begin{aligned} \text{workdone} &= \text{force} \times \text{distance} \\ &= 45\cos 14^\circ \times 25 \\ &= 1091.58 \approx 1090 \text{ J} \quad \textbf{(Ans).} \end{aligned}$$

Question 12

A car of mass 1250 kg moves from the bottom to the top of a straight hill of length 500m. The top of the hill is 30 m above the level of the bottom. The power of the car's engine is constant and equal to 30 000 W. The car's acceleration is 4 ms^{-2} at the bottom of the hill and is 0.2 ms^{-2} at the top. The resistance to the car's motion is 1000 N. Find

(i) the car's gain in kinetic energy, [5]

(ii) the work done by the car's engine. [3]

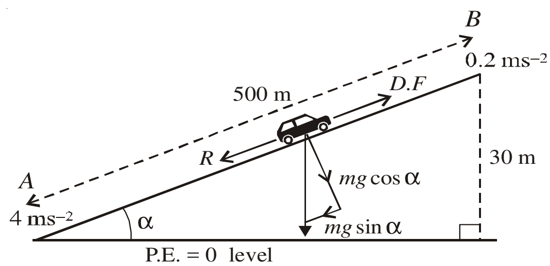
[N12/P4/Q6]

Suggested Solution:

(i) Given that, $m = 1250$ kg, $a_A = 4$ ms^{-2}, $a_B = 0.2$ ms^{-2}, $P = 30000$ W,

resistance, $R = 1000$ N, $\sin\alpha = \dfrac{30}{500} = 0.06$

Driving force at A, $D.F_A = \dfrac{P}{v_A} = \dfrac{30000}{v_A}$



Applying Newton's 2nd law

$D.F - mg\sin\alpha - R = ma_A$

$\Rightarrow \dfrac{30000}{v_A} - (1250)(10)(0.06) - 1000 = 1250(4)$

$\Rightarrow \dfrac{30000}{v_A} = 6750 \Rightarrow v_A = 4.44 \text{ ms}^{-1}$

Driving force at B, $D.F_B = \dfrac{P}{v_B} = \dfrac{30000}{v_B}$

Applying Newton's 2nd law

$D.F - mg\sin\alpha - R = ma_B$

$\Rightarrow \dfrac{30000}{v_B} - (1250)(10)(0.06) - 1000 = 1250(0.2)$

$\Rightarrow \dfrac{30000}{v_B} = 2000 \Rightarrow v_B = 15 \text{ ms}^{-1}$

$$\begin{aligned} \text{Gain in kinetic energy} &= \text{K.E}_B - \text{K.E}_A \\ &= \frac{1}{2}mv_B^2 - \frac{1}{2}mv_A^2 \\ &= \frac{1}{2}(1250)(15^2 - 4.44^2) = 128304 \text{ J} \approx 128 \text{ kJ} \quad \textbf{(Ans).} \end{aligned}$$

(ii) Using work-energy relation,

work done by net force = change in mechanical energy

$\Rightarrow$ W.D by driving force − W.D against resistance = gain in K.E. + gian in P.E.

$\Rightarrow$ W.D by car's engine − (1000)(500) = 128304 + (1250)(10)(30)

$\Rightarrow$ W.D by car's engine − 500000 = 503304

$\Rightarrow$ W.D by car's engine = 1003304 ≈ 1000 kJ (3 sf) **(Ans).**

Question 13

A and B are two points 50 metres apart on a straight path inclined at an angle θ to the horizontal, where $\sin\theta = 0.05$, with A above the level of B. A block of mass 16 kg is pulled down the path from A to B. The block starts from rest at A and reaches B with a speed of 10 ms^{-1}. The work done by the pulling force acting on the block is 1150 J.

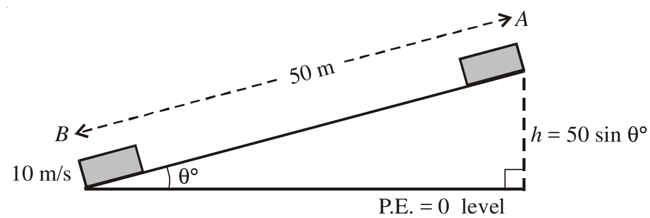
(i) Find the work done against the resistance to motion. [3]

The block is now pulled up the path from B to A. The work done by the pulling force and the work done against the resistance to motion are the same as in the case of the downward motion.

(ii) Show that the speed of the block when it reaches A is the same as its speed when it started at B. [2]

[J13/P4/Q2]

Suggested Solution:



(i) Gain in K.E. at $B = \frac{1}{2}mv^2$

$$= \frac{1}{2}(16)(10)^2 = 800 \text{ J}$$

Loss in P.E. at $B = mgh$

$$= (16)(10)(50\sin\theta)$$

$$= (16)(10)(50 \times 0.05) = 400 \text{ J}$$

using work-energy relation,

Work done by net force = Change in mechanical energy

$\Rightarrow$ W.D by pulling force – W.D against resistance = gain in K.E – loss in P.E.

$\Rightarrow$ 1150 – W.D against resistance = 800 – 400

$\Rightarrow$ W.D against resistance = 1150 – 400 = 750 J **(Ans).**

(ii) K.E. at $B = (\frac{1}{2}mv_B^2)$ J, P.E. at $B = 0$ J

total mechanical energy at B = K.E. + P.E

$$= \frac{1}{2}mv_B^2 + 0 = (\frac{1}{2}mv_B^2) \text{ J}$$

K.E. at $A = (\frac{1}{2}mv_A^2)$ J, P.E. at $A = 400$ J

total mechanical energy at $A = (\frac{1}{2}mv_A^2 + 400)$ J

using work-energy relation,

W.D by pulling force – W.D against resistance = Change in mechanical energy

$$1150 - 750 = (\frac{1}{2}mv_A^2 + 400) - (\frac{1}{2}mv_B^2)$$

$$400 = \frac{1}{2}mv_A^2 + 400 - \frac{1}{2}mv_B^2$$

$$\frac{1}{2}mv_A^2 = \frac{1}{2}mv_B^2$$

$$v_A^2 = v_B^2$$

$$v_A = v_B \quad \textbf{(Shown).}$$

Question 14

A car of mass 1000 kg is travelling on a straight horizontal road. The power of its engine is constant and equal to P kW. The resistance to motion of the car is 600 N. At an instant when the car's speed is 25 ms^{-1}, its acceleration is 0.2 ms^{-2}. Find

(i) the value of P, [4]

(ii) the steady speed at which the car can travel. [3]

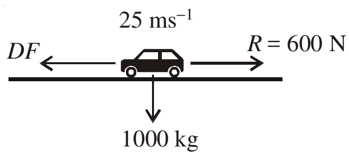
[J13/P4/Q5]

Suggested Solution:

(i) Given that,

$m = 1000$ kg, $v = 25$ m/s, $R = 600$ N, $a = 0.2$ m/s^2, power $= P$ kW $= 1000P$ W

Driving force, $DF = \dfrac{\text{power}}{\text{velocity}} = \dfrac{1000P}{25} = 40P$



applying Newton's 2nd law,

$$\text{Net force} = ma$$
$$DF - R = ma$$
$$40P - 600 = 1000(0.2)$$
$$40P = 800 \quad \Rightarrow \quad P = 20 \text{ W} \quad \textbf{(Ans).}$$

(ii) At steady speed (or maximum speed), acceleration, $a = 0$

$\Rightarrow \quad DF = R$

$\Rightarrow \quad \dfrac{1000(20)}{v} = 600 \quad \Rightarrow \quad v = \dfrac{20000}{600} = 33.3$

$\therefore$ steady speed $= 33.3$ ms^{-1} **(Ans).**

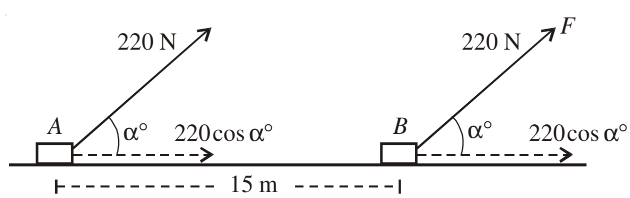
Question 15

A box of mass 25 kg is pulled in a straight line along a horizontal floor. The box starts from rest at a point A and has a speed of 3 ms^{-1} when it reaches a point B. The distance AB is 15 m. The pulling force has magnitude 220 N and acts at an angle of $\alpha°$ above the horizontal. The work done against the resistance to motion acting on the box, as the box moves from A to B, is 3000 J. Find the value of α. [5]

[N13/P4/Q2]

Suggested Solution:

K.E. at $A = \dfrac{1}{2}(25)(0)^2 = 0,$



K.E. at $B = \dfrac{1}{2}(25)(3)^2 = 112.5$ J

Driving force, $DF = 220\cos\alpha,$

Using work-energy relation,

work done by net force $=$ change in K.E.

$\Rightarrow$ W.D by driving force $-$ W.D against resistance $=$ K.E. at $B -$ K.E. at A

$\Rightarrow \quad (220\cos\alpha)(15) - 3000 = 112.5 - 0$

$\Rightarrow \quad 3300\cos\alpha = 3112.5$

$\Rightarrow \quad \cos\alpha = 0.9432 \quad \Rightarrow \quad \alpha = 19.4° \quad$ **(Ans).**

Learning CORNER

Question 46

A particle of mass 1.6 kg is dropped from a height of 9 m above horizontal ground. The speed of the particle at the instant before hitting the ground is 12 m s^{-1}.

Find the work done against air resistance. [3]

[J23/P4/Q1]

Suggested Solution:

Speed at $A=0$, Speed at $B=12\ \text{ms}^{-1}$

Using work-energy relation,

Work done by net force = Change in mechanical energy

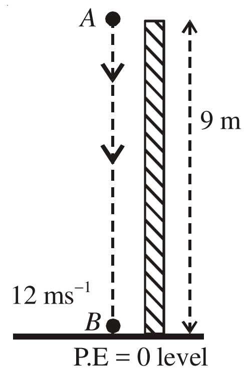
$\Rightarrow$ W.D by driving force − W.D against resistance $= M.E_B - M.E_A$

$\Rightarrow$ W.D against resistance = W.D by driving force $-(M.E_B - M.E_A)$

$\Rightarrow$ W.D against resistance $= 0-\left[(\text{P.E}_B+\text{K.E}_B)-(\text{P.E}_A+\text{K.E}_A)\right]$

$\Rightarrow$ W.D against resistance $= -\left(0+\frac{1}{2}(1.6)(12)^2\right)+\left((1.6)(10)(9)+0\right)$

$= -115.2+144 = 28.8\ \text{J}$ **(Ans).**



Question 47

An athlete of mass 84 kg is running along a straight road.

(a) Initially the road is horizontal and he runs at a constant speed of 3 m s^{-1}. The athlete produces a constant power of 60 W.

Find the resistive force which acts on the athlete. [1]

(b) The athlete then runs up a 150 m section of the road which is inclined at 0.8° to the horizontal. The speed of the athlete at the start of this section of road is 3 m s^{-1} and he now produces a constant driving force of 24 N. The total resistive force which acts on the athlete along this section of road has constant magnitude 13 N.

Use an energy method to find the speed of the athlete at the end of the 150 m section of road. [6]

[J23/P4/Q4]

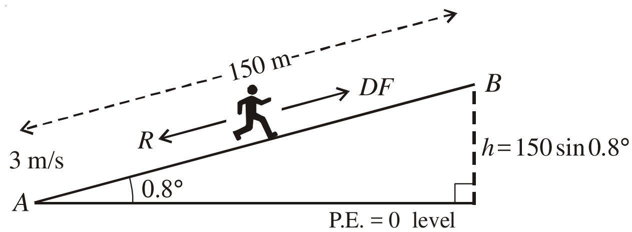
Suggested Solution:

(a) Driving force, $DF = \dfrac{\text{power}}{\text{velocity}} = \dfrac{60}{3} = 20\ \text{N}$

at constant speed, DF = resistance force

$\therefore$ Resistance force = 20 N **(Ans).**

(b) Let the speed of athlete at the end of road section be v_B.



K.E at $A = \frac{1}{2}(84)(3)^2 = 378$ J,

P.E at $A = 0$

$\therefore$ Total mechanical energy at $A = 378$ J

K.E at $B = \frac{1}{2}(84)(v_B)^2 = 42(v_B)^2$

P.E at $B = (84)(10)(150\sin 0.8) = 1759$ J

$\therefore$ Total mechanical energy at $B = 42(v_B)^2 + 1759$

Work done by driving force $= 24\times 150 = 3600$ J

Work done against resistance $= 13\times 150 = 1950$ J

Using work-energy relation,

work done by net force = change in mechanical energy

$\Rightarrow$ W.D by driving force − W.D against resistance = M.E at B − M.E at A

$\Rightarrow$ $3600 - 1950 = 42(v_B)^2 + 1759 - 378$

$\Rightarrow$ $1650 = 42(v_B)^2 + 1381$

$\Rightarrow$ $42(v_B)^2 = 269 \quad \Rightarrow \quad v_B = \sqrt{\frac{269}{42}} = 2.53 \text{ ms}^{-1}$ **(Ans).**

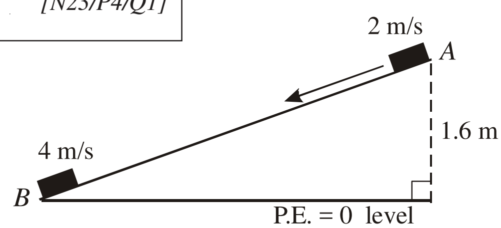
Question 48

A block of mass 15 kg slides down a line of greatest slope of an inclined plane. The top of the plane is at a vertical height of 1.6 m above the level of the bottom of the plane. The speed of the block at the top of the plane is 2 m s^{-1} and the speed of the block at the bottom of the plane is 4 m s^{-1}.

Find the work done against the resistance to motion of the block. [4]

[N23/P4/Q1]

Suggested Solution:



Speed at $A = 2 \text{ ms}^{-1}$, Speed at $B = 4 \text{ ms}^{-1}$

Using work-energy relation,

Work done by net force = Change in mechanical energy

$\Rightarrow$ W.D by driving force − W.D against resistance = $M.E_B - M.E_A$

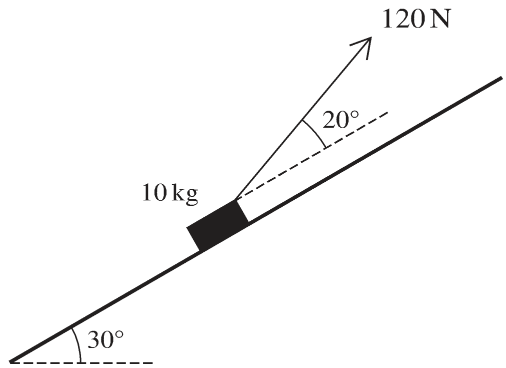
$\Rightarrow$ W.D against resistance = W.D by driving force − $(M.E_B - M.E_A)$

$\Rightarrow$ W.D against resistance $= 0 - \left[(\text{P.E}_B + \text{K.E}_B) - (\text{P.E}_A + \text{K.E}_A)\right]$

$\Rightarrow$ W.D against resistance $= -\left(0 + \frac{1}{2}(15)(4)^2\right) + \left((15)(10)(1.6) + \frac{1}{2}(15)(2)^2\right)$

$= -120 + 240 + 30 = 150$ J **(Ans).**

Question 49

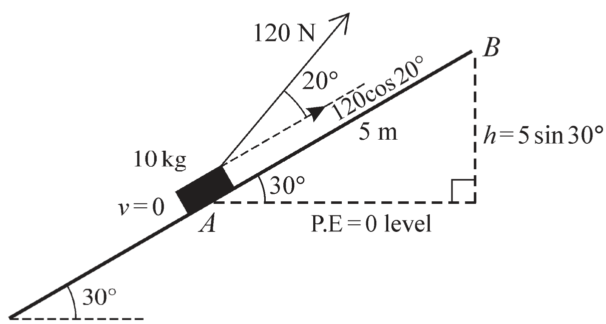


A block of mass 10 kg is at rest on a rough plane inclined at an angle of 30° to the horizontal. A force of 120 N is applied to the block at an angle of 20° above a line of greatest slope (see diagram). There is a force resisting the motion of the block and 200 J of work is done against this force when the block has moved a distance of 5 m up the plane from rest.

Find the speed of the block when it has moved a distance of 5 m up the plane from rest. [5]

[N23/P4/Q3]

Suggested Solution:



K.E at $A = 0$, P.E at $A = 0$

$\therefore$ Total mechanical energy at $A = 0$ J

K.E at $B = \frac{1}{2}(10)v^2 = 5v^2$

P.E at $B = (10)(10)(5\sin 30°) = 250$ J

$\therefore$ Total mechanical energy at $B = 5v^2 + 250$

Work done by driving force $= 5 \times 120\cos 20°$

$= 563.816$ J

Work done against resistance $= 200$ J

Using work-energy relation,

W.D by driving force − W.D against resistance = Mechanical energy at B − Mechanical energy at A

$\Rightarrow\ 563.816 - 200 = 5v^2 + 250 - 0$

$\Rightarrow\ 113.816 = 5v^2$

$\Rightarrow\ v^2 = \dfrac{113.816}{5}$

$\Rightarrow\ v = \sqrt{\dfrac{113.816}{5}} = 4.77\ \text{ms}^{-1}$ **(Ans).**

Alternative Solution

Resistance force, $R = \dfrac{\text{WD against resistance}}{\text{Distance}}$

$= \dfrac{200}{5} = 40 \text{ N}$

Driving Force, $DF = 120\cos 20° = 112.76 \text{ N}$

Applying Newton's 2nd law,

$DF - 10g\sin 30° - R = ma$

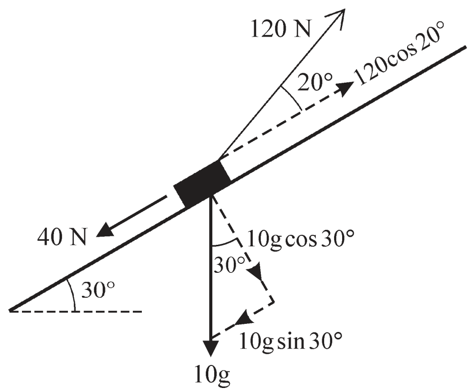
$\Rightarrow\ 112.76 - (10)(10)(0.5) - 40 = 10a$

$\Rightarrow\ 22.76 = 10a \ \Rightarrow\ a = 2.276 \text{ ms}^{-2}$

Using $v^2 - u^2 = 2as$

$v^2 - 0 = 2(2.276)(5)$

$\Rightarrow\ v^2 = 22.76 \ \Rightarrow\ v = 4.77 \text{ ms}^{-1}$ **(Ans).**



Question 50

A railway engine of mass 120 000 kg is towing a coach of mass 60 000 kg up a straight track inclined at an angle of α to the horizontal where $\sin\alpha = 0.02$. There is a light rigid coupling, parallel to the track, connecting the engine and coach. The driving force produced by the engine is 125 000 N and there are constant resistances to motion of 22 000 N on the engine and 13 000 N on the coach.

(a) Find the acceleration of the engine and find the tension in the coupling. [5]

At an instant when the engine is travelling at 30 m s^{-1}, it comes to a section of track inclined upwards at an angle β to the horizontal. The power produced by the engine is now 4 500 000 W and, as a result, the engine maintains a constant speed.

(b) Assuming that the resistance forces remain unchanged, find the value of β. [4]

[N23/P4/Q6]

Suggested Solution:

(a) Consider the forces acting on the Engine.

Applying Newton's 2nd law,

$DF - R - mg\sin\alpha - T = ma$

$\Rightarrow\ 125000 - 22000 - 120000(10)(0.02) - T = 120000a$

$\Rightarrow\ 125000 - 22000 - 24000 - T = 120000a$

$\Rightarrow\ 79000 - T = 120000a$

$\Rightarrow\ T = 79000 - 120000a$ .... (1)

Learning CORNER

Consider the forces acting on the Coach.

Applying Newton's 2nd law,

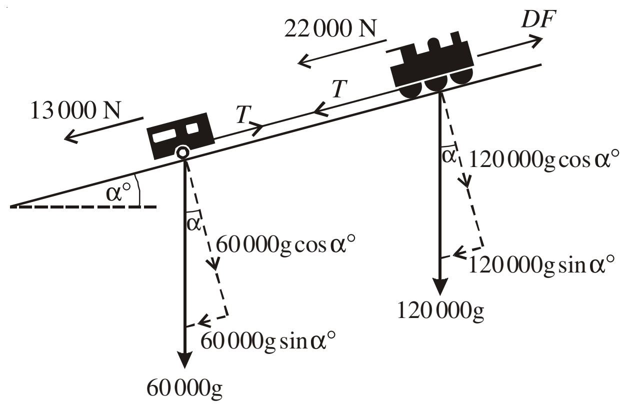
$T - R - mg\sin\alpha = ma$

$\Rightarrow\ T - 13000 - 60000(10)(0.02) = 60000a$

$\Rightarrow\ T - 13000 - 12000 = 60000a$

$\Rightarrow\ T - 25000 = 60000a$

$\Rightarrow\ T = 60000a + 25000$ .... (2)



Solving Equations (1) & (2) simultaneously,

$79000 - 120000a = 60000a + 25000$

$\Rightarrow\ 79000 - 25000 = 60000a + 120000a$

$\Rightarrow\ 54000 = 180000a$

$\Rightarrow\ a = \dfrac{54000}{180000} \quad \Rightarrow \quad a = 0.3 \text{ ms}^{-2}$ **(Ans).**

Substitute value of a into eq. (2),

$T = 60000(0.3) + 25000 \quad \Rightarrow \quad T = 43000 \text{ N}$ **(Ans).**

Alternative Solution

By treating Engine and Coach as one system,

Driving force, $DF = 125000$ N

Total mass $= 120000 + 60000 = 180000$ kg

Total resistance to motion

$= 22000 + 13000 + 120000g\sin\alpha + 60000g\sin\alpha$

$= 35000 + (120000)(10)(0.02) + (60000)(10)(0.02)$

$= 35000 + 24000 + 12000 = 71000$ N

Applying Newton's 2nd law,

$DF - R = ma$

$125000 - 71000 = 180000a$

$54000 = 180000a$

$a = 0.3 \text{ ms}^{-2}$ **(Ans).**

Now, consider the forces acting on the Coach.

Applying Newton's 2nd law,

$T - R - mg\sin\alpha = ma$

$\Rightarrow\ T - 13000 - 60000(10)(0.02) = 60000(0.3)$

$\Rightarrow\ T - 13000 - 12000 = 18000$

$\Rightarrow\ T - 25000 = 18000$

$\Rightarrow\ T = 43000$ N **(Ans).**

(b) Considering Engine and the Coach as one body,

Driving force, $DF = \dfrac{P}{v} = \dfrac{4500000}{30} = 150000 \text{ N}$

$$\begin{aligned}\text{Total resistance to motion} &= 22000 + 13000 + 120000g\sin\beta + 60000g\sin\beta \\ &= 35000 + 120000(10)\sin\beta + 60000(10)\sin\beta \\ &= 35000 + 1200000\sin\beta + 600000\sin\beta \\ &= 35000 + 1800000\sin\beta\end{aligned}$$

At constant speed,

$DF =$ Total Resistance to motion

$\Rightarrow\ 150000 = 35000 + 1800000\sin\beta$

$\Rightarrow\ 1800000\sin\beta = 115000$

$\Rightarrow\ \sin\beta = \dfrac{115000}{1800000} \quad \Rightarrow \quad \beta = 3.66° \approx 3.7°$ **(Ans).**

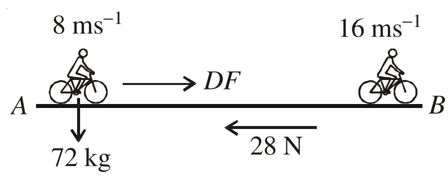
Question 51

A cyclist and bicycle have a total mass of 72 kg. The cyclist rides along a horizontal road against a total resistance force of 28 N.

Find the total work done by the cyclist to increase his speed from 8 m s^{-1} to 16 m s^{-1} while travelling a distance of 100 metres. [3]

[J24/P4/Q1]

Suggested Solution:



Given that, $m = 72$ kg, resistance, $R = 28$ N, $s = 100$ m,

$v_A = 8 \text{ ms}^{-1}, \quad v_B = 16 \text{ ms}^{-1}$

Using work-energy relation,

Work done by net force = change in K.E.

$\Rightarrow$ W.D by driving force − W.D against resistance = K.E. at B − K.E. at A

$\Rightarrow$ W.D by cyclist $-(28)(100) = \dfrac{1}{2}(72)(16)^2 - \dfrac{1}{2}(72)(8)^2$

$\Rightarrow$ W.D by cyclist $-2800 = \dfrac{1}{2}(72)\left(16^2 - 8^2\right)$

$\Rightarrow$ W.D by cyclist $= 36(192) + 2800$

$= 9712$ J **(Ans).**

Question 52

A car has mass 1400 kg. When the speed of the car is v m s^{-1} the magnitude of the resistance to motion is kv^2 N where k is a constant.

(a) The car moves at a constant speed of 24 m s^{-1} up a hill inclined at an angle of α to the horizontal where $\sin\alpha = 0.12$. At this speed the magnitude of the resistance to motion is 480 N.

(i) Find the value of k. [1]

(ii) Find the power of the car's engine. [3]

(b) The car now moves at a constant speed on a straight level road. Given that its engine is working at 54 kW, find this speed. [3]

[J24/P4/Q4]

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Suggested Solution:

(a) (i) Given that, resistance, $R = 480$ N

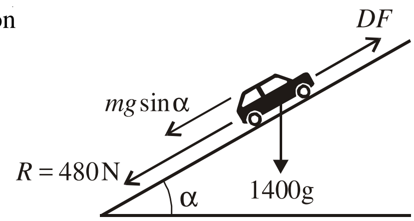
$\Rightarrow \ kv^2 = 480$

$\Rightarrow \ k(24)^2 = 480$

$\Rightarrow \ k = \dfrac{480}{24^2} \ \Rightarrow \ k = \dfrac{480}{576} \ \Rightarrow \ k = \dfrac{5}{6}$ **(Ans).**

(ii) At constant speed, Driving force = Resistance to motion

Resolving the forces along the plane,



$\nearrow$: $DF = R + mg\sin\alpha$

$DF = 480 + (1400)(10)(0.12)$

$DF = 2160$ N

Now, Power $= DF \times v$

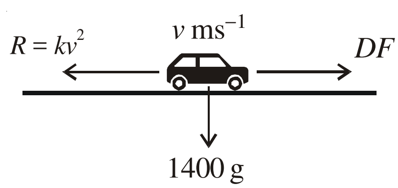
$= (2160)(24)$

$= 51840$ W. **(Ans).**

(b) Resistance to motion, $R = kv^2$,

Driving force, $DF = \dfrac{P}{v}$

At constant speed, $DF = R$



$\Rightarrow \ \dfrac{P}{v} = kv^2$

$\Rightarrow \ \dfrac{54000}{v} = \dfrac{5}{6}v^2$

$\Rightarrow \ 54000 \times \dfrac{6}{5} = v^3$

$\Rightarrow \ v^3 = 64800 \ \Rightarrow \ v = 40.2 \text{ ms}^{-1}$ **(Ans).**

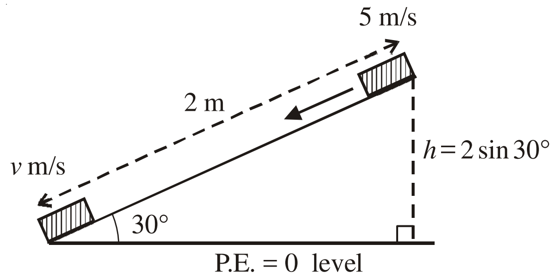
Question 53

A block of mass 20 kg is held at rest at the top of a plane inclined at 30° to the horizontal. The block is projected with speed 5 ms^{-1} down a line of greatest slope of the plane. There is a resistance force acting on the block. As the block moves 2 m down the plane from its point of projection, the work done against this resistance force is 50 J.

Find the speed of the block when it has moved 2 m down the plane. [4]

[N24/P4/Q2]

Suggested Solution:



K.E. at the top $= \frac{1}{2}(20)(5)^2 = 250$ J,

K.E. at bottom $= \frac{1}{2}(20)(v)^2 = 10v^2$

$\therefore$ Gain in K.E. $= (10v^2 - 250)$ J

P.E. at the top $= mgh$

$= (20)(10)(2\sin 30°) = 200$ J

P.E. at bottom = 0 J

$\therefore$ Loss in P.E. $= 200 - 0 = 200$ J

Using work-energy relation,

Loss in P.E = gain in K.E + work done against resistance

$$200 = 10v^2 - 250 + 50$$

$$200 = 10v^2 - 200$$

$$10v^2 = 400$$

$$v^2 = 40 \quad \Rightarrow \quad v = 6.32\ \text{ms}^{-1}$$ **(Ans).**